Expert Systems with Applications 39 (2012) 7690–7700

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Dynamic estimation of markets exhibiting a prey–predator behavior

Christos Michalakelis *, Charisios Christodoulos, Dimitrios Varoutas, Thomas Sphicopoulos

National and Kapodistrian University of Athens, Panepistimioupolis, Illisia, Athens, Greece

ARTICLE INFO

Keywords:
Market structure
Market competition
Market shares
Telecommunications forecasting
Prey–predator species
Lotka–Volterra model

ABSTRACT

The evolution of market concentration in high technology saturated markets with a dominant player is dynamically estimated, based on concepts of population biology. The mathematical description was performed using the Lotka–Volterra model and the corresponding parameters were estimated by genetic algorithms. The proposed methodology shows itself capable of estimating market equilibrium and market concentration, the latter expressed by corresponding market shares. Evaluation of the presented methodology in the area of fixed lines telecommunications market led to accurate results, as compared to historical data, in a specific case study. This methodology can provide valuable inputs for managerial decisions, strategic planning and regulatory decisions to the players of a high technology market.

1. Introduction

Telecommunications are related with the biggest and most significant civil investment regarding new technologies and services. For a long period of time they used to operate under governmental provision and monopolistic regime, mainly because of governmental strategic perceptions and military security. During this time, the telecommunications industry was a relatively stable environment (van Kranenburg & Hagedoorn, 2008), since incumbent telecommunications operators offered only traditional voice and data transfer services via fixed telecommunications networks.

The liberalization of basic fixed telephone services started during the mid-1990s. By 1999, competition for such services had become relatively common, although it was still not the policy of a majority of countries. During 1999 more than 40% of the countries in Europe and 35% of the countries in the Americas permitted competition for basic fixed services (Banerjee & Ros, 2004). However, only 22% of Asian, 15% of the Middle East, and 15% of African countries had followed. In 2000, competition for basic fixed services was still restricted in 105 countries. As far as Europe is concerned, since the 1990s the European Union (EU), in the context of its plans for a Single European Market, has been pursuing a common telecommunications regulatory policy aiming to establish a liberalized and harmonized pan-European telecommunications market, to stimulate economic growth, increase employment and the standard of living in the European Community. Towards this direction, the Commission of the European Community in the Green Paper (European Commission, 1987) has put forward the principles for the liberalization of the telecommunications sector and described the common policy, in view of the single market. The main goal of market opening and restructuring was to promote market structures that would enable the exploitation of substantial demand and innovation potentials in the communications industry. The initial regulation aimed to transform telecommunication monopolies into competitive industries. Since the 1st of January 1998 almost all of the telecommunication markets in the EU have been fully liberalized, although full liberalization was delayed in Portugal and Greece until 2000 and 2001, respectively (Grzybowski, 2008). As stated in (van Kranenburg & Hagedoorn, 2008), where an informative review regarding the European telecommunications industry can be found, "*The liberalization of European telecommunications markets and the privatization of many historically state-owned telecommunications companies are probably the two most significant changes in the landscape*".

Obviously, these changes amount to a substantially new worldwide environment for market players in the telecommunications area and present new challenges in terms of both new opportunities and new competitors. According to the contemporary economic theory regarding competition, (Baye, 2006; Shy, 1995) downward pressure on the prices of telecommunication services is likely to lead service providers to introduce new pricing packages in order to better satisfy consumer preferences. This was exactly the case when liberalization started in the new converged telecommunication market, as a fierce competition emerged between the incumbent operators and the new companies that appeared, all trying to increase the number of their customers by providing services and products in very attractive prices. This is regarded as a significant dimension of the market structure, since it plays an important part in determining market power and therefore business behavior and performance in a market that was

* Corresponding author. Tel.: +30 210 7275236.
E-mail addresses: michalak@di.uoa.gr (C. Michalakelis), christod@di.uoa.gr (C. Christodoulos), arkas@di.uoa.gr (D. Varoutas), thomas@di.uoa.gr (T. Sphicopoulos).

0957-4174/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.eswa.2012.01.049

reformed from a monopolistic to oligopolistic or even to a more competitive structure.

Studying the concentration of the new market schema is therefore necessary, in order to identify its possible peculiarities, to describe competitors' behavior and to provide necessary inputs to legislation and regulation authorities, regarding market structure and the degree of competition (Curry & George, 1983; Marfels, 1972; Saving, Feb., 1970; Tirole, 1988), as well as predictions for the future regarding, among others, potential entry of new providers (Baye, 2006; Shy, 1995). Moreover, market concentration, as expressed by the values of market shares, is of major interest for providers as well, since it is strongly related to managerial decisions, available actions to be taken and expectations towards competition. Such decisions are usually accompanied by investments and business plans, targeting to enhance the ability of providers to meet the market's demand. Since market shares reflect, among others, the influence of providers' pricing policies over the subscribers, this is another important aspect to be taken under consideration.

In the context of the present work, the evolution of fixed telephony market (PSTN) concentration is studied, based on the evolution of the corresponding market shares of both the incumbent and the alternative providers, by adopting approaches from evolutionary theory of population biology and population dynamics. More specifically, market evolution is estimated and forecasted by applying the Lotka–Volterra model that describes the interaction of species in a prey–predator mode (Murray, 2002; Neal, 2004). Lotka–Volterra models have already been used in literature in order to model market dynamics, mainly in a duopolistic market (Kim, Lee, & Ahn, 2006; Lopez & Sanjuan, 2001; Tschirhart, 2000; Wijeratne, Yi, & Wei, 2007), or to provide forecasts for the stock market (Lee, Lee, & Oh, 2005; Modis, 1999).

1.1. Research objectives and contribution

The above approaches, although they constitute important contributions to the literature, they refer to markets where competition is already established. The main objective of the present work is to fill this gap by proposing a methodology that can describe the dynamics of an already saturated monopolistic market, which makes its first steps towards competition. Such an analysis is expected to provide valuable information to the existing competitors, as well as to the potential entrants and regulators, regarding market structure, degree of competition and market equilibrium.

The second objective is to point out the relation of pricing schemes with the corresponding market shares. Indeed, the call charges paid by the consumers influence, up to a certain point, their decision to switch to another provider, or to stay with their present one, thus influencing the share of each competitor into the market of reference.

The third objective is the application of the proposed methodology over a number of historical data series, in order to evaluate its performance and its ability to provide accurate estimations and reliable forecasts. Therefore, the evaluation of the proposed methodology was performed over historical data regarding market shares between the incumbent operator and the alternatives, for local, national long distance and international calls in five European countries. The data used for this evaluation were extracted from Eurostat's database (http://ec.europa.eu/eurostat/).

Accomplishment of the work's objectives would provide significant contribution to both research and practice. Research would benefit by the provision of new directions towards the development of a framework that incorporates the theories of population dynamics in order to analyze and study market competition, especially for high technology markets. For practitioners, the research findings can be very useful to strategic planning and decision making, in a continuously increased competitive environment, since more accurate a-priori estimates of the market dynamics patterns can be derived, including market equilibria and the estimation of the level of customers' switching among providers.

1.2. Population dynamics

Population dynamics is the study of marginal and long-term changes in the numbers (Neal, 2004), individual weights and age composition of individuals in one or several populations, and biological and environmental processes influencing those changes. The corresponding population modeling is an application of statistical models to the study of these changes in populations as a consequence of interactions of organisms with the physical environment, with individuals of their own species (intraspecies competition), and with organisms of other species (interspecies competition). Finally, one of the most important questions population modeling seeks to answer is if interacting species can coexist or not and what are the major factors that affect coexistence.

The rest of the paper is structured as follows: Section 2 is devoted in modeling of the fixed lines market dynamics according to the population biology approach. Section 3 provides a short overview of the mathematical concepts of population dynamics, especially the dynamics of the prey–predator system. Based on these concepts, the development of the proposed methodology is presented in Section 4, and the corresponding case study results are presented in Section 5. In Section 6 the methodology's forecasting ability is evaluated and finally, Section 7 describes an overview and the conclusions of the work conducted in this paper, together with directions for future work.

2. Approaching fixed line market dynamics by population biology modeling

The hypothesis concerning the variation of population is that the rate of its change is proportional to the current size of the population and the most common approach for modeling population growth of a species, in the absence of any competitors is given by (Boyce & DiPrima, 2005; Neal, 2004)

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) \tag{1}$$

In Eq. (1) $N(t)$ is the size of population at time t, the constant r is the growth rate, and K is the saturation level or the environmental carrying capacity, for the given species. K is the upper bound that is reached, but not exceeded, by growing populations starting below this value. Models based on the above approach are widely used in modern literature for demand estimation and forecasting, such as the logistic family growth models (Bewley & Fiebig, 1988; Fisher & Pry, 1971) and the Gompertz model (Gompertz, 1825; Rai, 1999). An application of these demand models over mobile telephony diffusion set can be found in (Michalakelis, Varoutas, & Sphicopoulos, 2008).

However, when more than one species coexist in the same environment, they are expected to interact between each other in a number of ways. Definitions and descriptions of species interaction can be found in (Begon, Townsend, & Harper, 2006, 4th ed.; Neal, 2004) and a more precise definition, regarding interaction of species, is given in (Murray, 2002), where three types of interaction are identified: (i) If the growth rate of one population is decreased and the other increased the populations are in a "*prey–predator*" situation. (ii) If the growth rate of each population is decreased then it is "*competition*". (iii) If each population's growth rate is enhanced then it is called "*mutualism*" or "*symbiosis*".

Towards this direction, the present work is based on the rationale of viewing the incumbent and the alternative operators as biological species competing for survival and growth, expressed in terms of market shares. The main target is to model the evolution of the market during the first time period that followed liberalization, when the incumbent operators dominated the market, in the absence of any competitors.

As soon as liberalization took place the alternative operators that emerged did not have their own network infrastructure, so they were depended on the incumbent. Moreover, by the time of liberalization, the fixed lines market was an already saturated one and since no further growth was expected all of the competitors would have to compete for a market share in an already mature market. Thus, the incumbent operator met a decrease in its market share, while the alternative operators enjoyed an increase in their corresponding shares and growth. Moreover, convergence of the telecommunication services accelerates redistribution of market shares, since the alternative operators provide not only voice services but also internet access or even triple play services, at low cost.

Thus, following the above, the prey–predator model is considered as the most appropriate approach for capturing the evolution of the telecommunications market, in terms of market shares. The incumbent operator is viewed as the prey and all the alternative operators as a predator, which bases its survival on consuming the prey's market share. Thus, the prey's population decreases, while the predator's increases. Market shares is a quite accurate indicator for estimating the degree of competition, since they can be considered as the observed outcome of the underlying, usually non-cooperative, game of the participating players–service providers. They reflect the results of managerial and strategic decisions, such as advertising, pricing policy, quality of services and more. In the next section, the mathematical model for describing the interaction of species in a prey–predator mode is presented, which turned out to adequately describe the fixed line market dynamics.

3. Population dynamics – prey predator species

The most common model for capturing the measure of interaction between species is the well-known Lotka–Volterra model, based on the work of Alfred J. Lotka and Vito Volterra. The approach is mainly based on incorporating suitable parameters into a corresponding system of differential equations. Analytical description together with informative examples regarding prey–predator interaction between species can be widely found in literature, such as in (Begon et al., 2006, 4th ed.; Boyce & DiPrima, 2005; Murray, 2002; Neal, 2004). Theoretical analyses together with applications of species interaction can be found in (Fay & Greeff, 2008; Freedman & Waltman, 1977, 1984, 1985; Kirlinger, 1989; Leach & Miritzis, 2006; Li & Yan, 2008; May & Leonard, 1975; Pao, 2003; Seifert, 1995; Shair & Tineo, 2007; Wangersky, 1978).

Construction of an interacting model consisting of one prey and one predator, whose populations at time t are denoted by $x(t)$ and $y(t)$, respectively, is based on the following assumptions:

- If no predator is present, population of the prey, at any time t, is described by Eq. (1).
- In the absence of the prey, the predator would die out at a rate proportional to its population.
- The number of encounters between the predator and the prey is proportional to the product of their populations. Each encounter tends to promote the growth of the predator and to inhibit the growth of the prey.

Following the above assumptions, the dynamics of the described system can be represented by the following system of first-order nonlinear differential equations:

$$\frac{dx(t)}{dt} = r \cdot x(t) \cdot \left(1 - \frac{x(t)}{K}\right) - b \cdot x(t) \cdot y(t) \quad (2)$$

$$\frac{dy(t)}{dt} = -c \cdot y(t) + d \cdot x(t) \cdot y(t)$$

In Eq. (2) $dx(t)/dt$, $dy(t)/dt$ are the rates of population change for the prey and the predator respectively and the parameters, r, K, b, c and d are all positive; parameter r stands for the growth rate of the prey, K for its carrying capacity (the maximum potential of the market) and b is a constant, used to capture the prey's extinction rate, as a result of interaction with the predator. Obviously, the greater the value of b the greater the effect of extinction of the prey, due to existence of the predator. Regarding the second equation that describes the predator, c is a constant that captures its death rate and d measures the predator's growth as a result of the interaction with the prey. Finally and according to the population biology terminology, the coefficients b and d measure the interspecies interacting effects (of each species over the other).

Given the two-dimensional autonomous system described by Eq. (2), every solution can be represented as a curve in the plane and each such solution curve is called a trajectory, or an orbit. The main objective is to determine the behavior of the solutions of the system, given the initial values for $x(t)$ and $y(t)$. This can be achieved by plotting a phase portrait, the geometric representation of the trajectories of a dynamical system, which shows how the qualitative behavior of the system is determined as x and y vary with the time t.

The above system has three critical points (or equilibrium solutions, the values of x and y for which the derivatives of system described by Eq. (2) becomes zero): $(0,0)$, $(K,0)$ and $\left(\frac{c}{d}, \frac{r}{b}\left(1 - \frac{c}{dK}\right)\right)$.

Since the system of Eq. (2) is a nonlinear system, linearization near critical points is necessary in order to study the qualitative behavior of the corresponding solution. Thus, for a nonlinear autonomous system described by:

$$\frac{dx}{dt} = P(x,y) \quad (3)$$

$$\frac{dy}{dt} = Q(x,y)$$

where P and Q are functions of x and y, having continuous partial derivatives up to order two, linearization at a critical point (x_0,y_0) is achieved by considering the following transformation, which moves the critical point to the origin:

$$X = x - x_0 \quad (4)$$

$$Y = y - y_0$$

After applying the above transformation and performing a Taylor series expansion the system of Eq. (3) becomes:

$$\frac{dX}{dt} = P(X + x_0, Y + y_0) = P(x_0,y_0) + X\left.\frac{\partial P}{\partial x}\right|_{x=x_0,y=y_0} + Y\left.\frac{\partial P}{\partial y}\right|_{x=x_0,y=y_0} + R_1(X,Y) \quad (5)$$

$$\frac{dY}{dt} = Q(X + x_0, Y + y_0) = Q(x_0,y_0) + X\left.\frac{\partial Q}{\partial x}\right|_{x=x_0,y=y_0} + Y\left.\frac{\partial Q}{\partial y}\right|_{x=x_0,y=y_0} + R_2(X,Y)$$

Since the nonlinear terms R_1 and R_2 satisfy the conditions:

$$\frac{R_1(X,Y)}{\sqrt{X^2+Y^2}} \to 0, \quad \frac{R_2(X,Y)}{\sqrt{X^2+Y^2}} \to 0 \quad (6)$$

as $\sqrt{X^2 + Y^2} \to 0$, they can be discarded. Also, $P(x_0, y_0) = Q(x_0, y_0) = 0$, since $(x_0, y_0)$is a critical point of the system of Eq. (3). Thus, the corresponding linearized system can be described by the following equations:

$$\frac{dX}{dt} = X\frac{\partial P}{\partial x}\bigg|_{x=x_0,y=y_0} + Y\frac{\partial P}{\partial y}\bigg|_{x=x_0,y=y_0} \tag{7}$$

$$\frac{dY}{dt} = X\frac{\partial Q}{\partial x}\bigg|_{x=x_0,y=y_0} + Y\frac{\partial Q}{\partial y}\bigg|_{x=x_0,y=y_0}$$

where as the corresponding Jacobian matrix is given by:

$$J(x_0, y_0) = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix}\bigg|_{x=x_0,y=y_0} \tag{8}$$

After linearization of the system, the analysis is concluded by applying Hartman's theorem (Hartman, 1964; Lynch, 2007) which states that if (x_0, y_0) is a critical point whose real part of the eigenvalues of the Jacobian matrix $J(x_0, y_0)$ are nonzero (hyperbolic critical point), then there is a neighborhood of this critical point on which the phase portrait of the nonlinear system can be adequately described by the phase portrait of the linearized system of Eq. (7).

4. Methodology for market shares evaluation

4.1. Definition of the model

As mentioned in the introductory section, construction of the proposed methodology was based on the main assumption of corresponding market share sizes of the competing providers, with an equivalent number of species interacting in a prey–predator mode. Indeed, this was the case immediately after liberalization, since the alternative operators that appeared prayed on the incumbent operator's market share, acting as predators. Therefore, the incumbent operator's market share started to decrease, while on the same time the alternatives' market shares increased, which if expressed in mathematical terms of Eq. (2) it is translated to $dx(t)/dt < 0$ and $dy(t)/dt < 0$. Indeed, immediately after liberalization, the alternative operators did not have their own network infrastructures thus they depended on the incumbent provider, which constituted their only "food supply". This is expected to be the situation until a market equilibrium will be met, which will probably happen when, some at least, alternative providers develop their own network infrastructures. In this case the system would be probably be described by the "competing species" formulation, of the Lotka–Volterra model.

Moreover, it is also assumed that all exterior factors, which may affect the dynamics of these species, such as regulation, price etc. are stable during the period under consideration.

Based on the above assumptions, the dynamics of the proposed system can be described by the system of Eq. (2), where $x(t)$ refers to the market share of the incumbent and $y(t)$ represents the market share of the alternative operators. However, in the case considered the carrying capacity for the prey – incumbent operator, in the absence of the predators would be equal to unity. Thus, setting $K = 1$, the initial system of Eq. (2) reduces to the following:

$$\frac{dx(t)}{dt} = r \cdot x(t) \cdot (1 - x(t)) - b \cdot x(t) \cdot y(t) \tag{9}$$

$$\frac{dy(t)}{dt} = -c \cdot y(t) + d \cdot x(t) \cdot y(t)$$

Analysis of the system of Eq. (9) includes calculation of the critical points and stability analysis for each one of them. Critical points are calculated at the equilibrium state of Eq. (9), by setting $dx(t)/dt = 0$ and $dy(t)/dt = 0$ and solving the corresponding system. Thus, the above system has the following three critical points: (0,0), (1,0) and $\left(\frac{c}{d}, \frac{r}{b}\left(1 - \frac{c}{d}\right)\right)$. Determination of stability is based on the calculation of the corresponding eigenvalues and eigenvectors at each critical point and examination of the nature of the eigenvalues (Boyce & DiPrima, 2005). In addition, since the first two critical points are related to situations that either both of the species extinct or only the incumbent survives, the focus is mainly set on the analysis of the third critical point, which corresponds to the survival of both species.

4.2. Case study description

Evaluation of the proposed methodology was performed over quarterly data, describing market shares in fixed lines phone calls over five European countries: Greece, Germany, Spain, United Kingdom and Ireland. In all the cases considered, evaluation was performed by considering two species, the incumbent operator and all the alternatives. Historical data were extracted from Eurostat's database and they correspond to local, national long distance and international calls, from the first quarter of year 2001 until the last quarter of year 2005.

The evaluation procedure is presented and explained in detail for the first case considered, Greece and since it was performed in the same way for all the participating countries, only the final results are provided for the rest cases, in corresponding subsections.

Actual market shares for the Greek fixed lines market are shown in Table 1. The first column refers to the quarter number (starting from 2001), the second to the incumbent operator's market share and the last one to the total market share of the alternative providers.

4.3. Estimation of the model parameters

In order to evaluate the effectiveness of the proposed model, the parameters of Eq. (9) should first be estimated. Such estimations are usually achieved by making reasonable assumptions based on the available data, but in the present methodology heuristic methods are employed by the means of genetic algorithms. Genetic algorithms are applied over a particular dataset in order to "train" the system, in terms of estimating the model parameters.

Genetic algorithms (GAs) were introduced by Holland (Goldberg, 1989; Holland, 1975) and they are adaptive heuristic search

Table 1
Market shares in fixed telecommunications in Greece (Source: Eurostat).

Quarter	OTE	Alternatives
1 (1st of 2001)	99.33	0.67
2	98.83	1.17
3	98.33	1.67
4	97.83	2.17
5	97.33	2.67
6	93.92	6.08
7	90.50	9.50
8	87.08	12.92
9	83.67	16.33
10	81.58	18.42
11	79.50	20.50
12	77.42	22.58
13	75.33	24.67
14	75.25	24.75
15	75.17	24.83
16	75.08	24.92
17	75.00	25.00
18	74.80	25.20
19	74.60	25.40
20	74.40	25.60

algorithms based on the mechanisms of natural selection and natural genetics. The basic concept of GAs is designed to simulate processes in natural systems necessary for evolution, specifically those that follow the principles first laid down by Charles Darwin of survival of the fittest. As such, they represent an intelligent exploitation of a random search within a defined search space to solve a problem. The key points to the process are reproduction, crossover and mutation, which are performed according to a given probability, just as it happens in real world. Reproduction involves copying (reproducing) solution vectors, crossover involves swapping partial solution vectors and mutation is the process of randomly changing a cell in the string of the solution vector preventing the possibility of the algorithm being trapped. The process continues until it reaches the optimal solution to the fitness function, which is used to evaluate individuals. Genetic algorithms have been applied for high technology demand estimation and they constitute a rapidly growing area of artificial intelligence (Michalakelis et al., 2008; Venkatesan & Kumar, 2002).

The general steps a genetic algorithm consists of are the following:

1. Define the fitness function, for the particular optimization problem.
2. Set crossover and mutation probabilities.
3. Randomly generate an initial population $N(0)$.
4. Generate $N(t+1)$ population by probabilistically selecting individuals from $N(t)$ to produce offsprings via genetic operators of crossover and mutation.
5. Compute fitness for each individual in the current population $N(t)$. Offsprings with values closer to the fitness function are more probable to contribute with one or more offsprings to the next generation. Offsprings that diverge from the fitness function are discarded.
6. Repeat steps 4 and 5 usually until, either a prefixed number of generations are created, or after some predefined time has elapsed.

In the context of the examined case study, the algorithm described above is performed, seeking to find the minimum value of the fitness function. In this case, the fitness function is defined to be the sum of the squared errors between the observed and the estimated market shares described by the system of Eq. (9). Convergence of the algorithm is expected to provide the values for the parameters that best describe the dynamics of the above process. Initial values for the algorithm were based on estimations of the rates of change of the market shares. Moreover, the algorithm was also executed with random initial values, in order to ensure that the algorithm will converge to the global minimum, instead of being trapped to a local one. In each case a number of about one million iterations were performed to ensure convergence. Application of the genetics algorithm for the particular optimization problem, based on the steps defined above, is illustrated in Fig. 1.

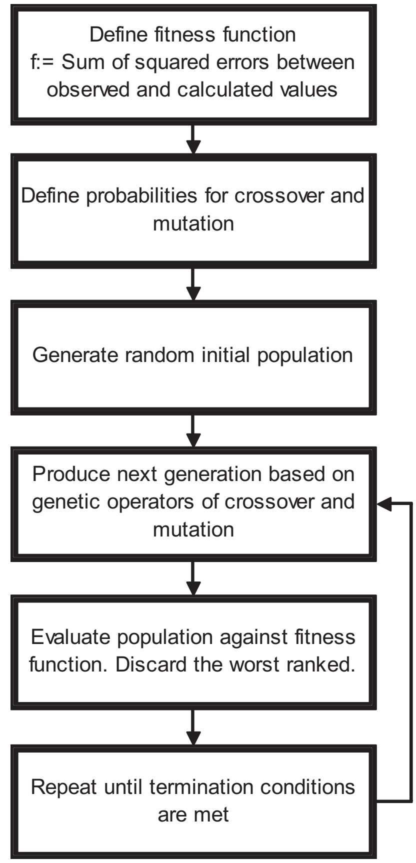

Fig. 1. Genetic algorithm procedure for the estimation of the proposed system's parameters.

5. Case study results

Application of the genetic algorithms described in the previous section provided with the following values for the corresponding parameters of the system of Eq. (9):

$$\frac{dx(t)}{dt} = 1.76x(t)(1 - x(t)) - 1.75x(t)y(t) \quad (10)$$

$$\frac{dy(t)}{dt} = -0.11y(t) + 0.23x(t)y(t)$$

Fitting of the estimated values over the observed is quite accurate, since the coefficient of determination (R^2) has a value of 0.985 and statistical measures were calculated at a value of 4.27E−04 for the Mean Squared Error (MSE) and a value of 0.07 for the Mean Absolute Percentage Error (MAPE). Fitting results are illustrated in Figs. 2 and 3.

Since $x(t)$ and $y(t)$ variables that appear in Eq. (10) correspond to market shares of the incumbent and the alternative operators, respectively, the estimated coefficients of the system provide quite important information regarding the dynamics of the described process. Thus, it can be observed that the rate at which the prey population decreases due to predation is quite high, almost as much as the rate of its growth. Therefore, if the system continues this way the incumbent operator is expected to meet a real threat by the alternatives. In addition, the growth rate of the predator due to interaction with the prey is quite higher than its death rate, which is in accordance with the previous finding, stating that the

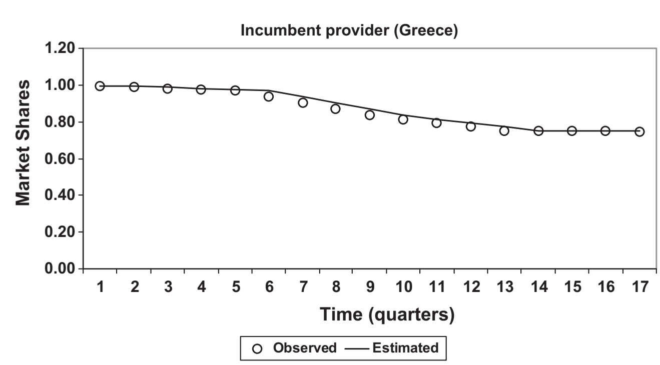

Fig. 2. Estimation results for incumbent operator's market shares.

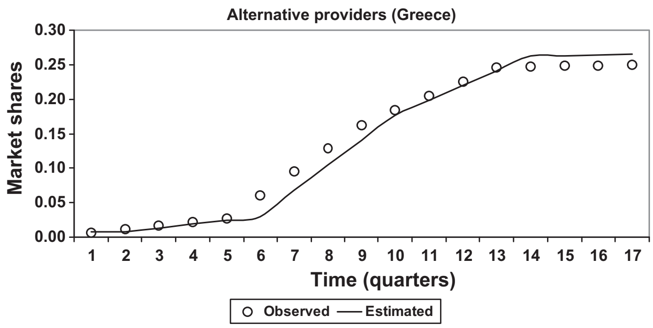

Fig. 3. Estimation results for alternative operators' market shares.

present system dynamics is very probable to result into a great reduction of the incumbent's market share with a simultaneous increase for the alternatives. These two parameters reflect the potential of the interspecies competition as they depict the dynamics of each competitor. They also provide quite useful information regarding the "churn effect" i.e. the movements of subscribers among the providers. Churn effect for each provider is depicted by the parameters' values that correspond to interspecies interaction. It is obvious that such kind of information, which can be derived by studying the present system, is an extremely helpful input in proceeding to critical managerial decisions.

As stated earlier, the stability analysis of the critical points is based on Hartman's theorem and is performed by resorting to the transformations described by Eq. (4), by considering the linearized system of Eq. (7) that approximates the system near a critical point (x_0, y_0) (Boyce & DiPrima, 2005; Lynch, 2007). Thus:

$$\frac{d}{dt}\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} r - 2rx_0 - by_0 & -bx_0 \\ dy_0 & -c + dx_0 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} \tag{11}$$

which, after substitution of the parameter values reduces to:

$$\frac{d}{dt}\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 1.76 - 3.52x_0 - 1.75y_0 & -1.75x_0 \\ 0.23y_0 & -0.11 + 0.23x_0 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} \tag{12}$$

The system has three solutions, (0,0), (1,0) and (0.478,0.522), all located at the positive quadrant. Plotting a direction field of the system (plot of tangent vectors to solutions of the system) provides a qualitative understanding of the behavior of the solutions. A direction field for the examined system is shown in Fig. 4. As observed, for the first two solutions, trajectories depart from the corresponding neighborhood, thus characterizing the solutions as unstable. On the other hand, all trajectories approach the third critical point validating the critical point as a stable node.

Mathematical analysis, based on eigenvalue analysis showed that the first two critical points (0,0) and (1,0) are indeed unstable, since the corresponding eigenvalues are positive. Regarding the third critical point $\left(\frac{c}{d}, \frac{r}{b}\left(1 - \frac{c}{d}\right)\right)$, substitution of the corresponding parameter values provides with an equilibrium value of 47.8% for the incumbent provider and 52.2% for the alternatives. This is an important finding since it states that, given the current process dynamics, the incumbent operator will cease dominance of the market. This finding can be explained based on the considerations of the introductory section. More specifically, subscribers of the alternative providers enjoy voice services for local, national and many international destinations at no extra cost, together with unlimited broadband access. As the quality of services of the alternative providers' increases, offering more services at lower prices, more subscribers tend to switch to them. This fact should be taken into account by the incumbent operator as a serious consideration, in order to meet the challenges of the new market.

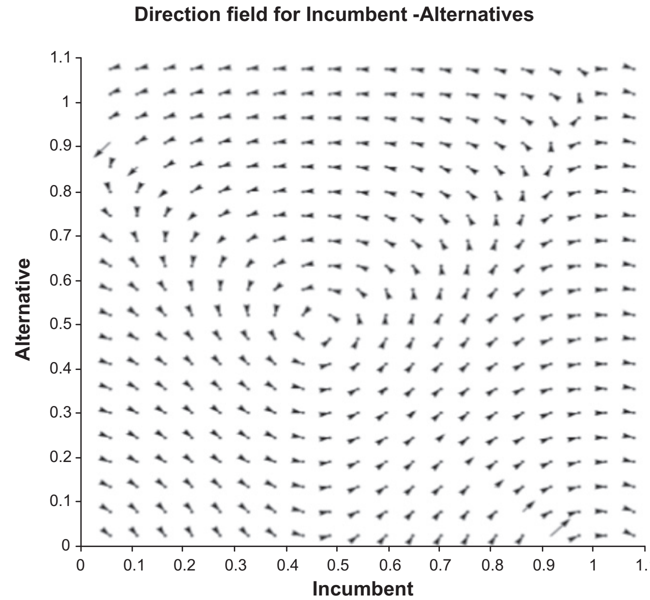

Fig. 4. Direction field for the prey–predator system of Eq. (10).

The stability of the last critical point is determined by substituting the corresponding values into Eq. (12):

$$\frac{d}{dt}\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} -0.84 & -0.84 \\ 0.12 & 0 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} \tag{13}$$

Eigenvalue analysis leads to the following general solution:

$$\begin{pmatrix} X \\ Y \end{pmatrix} = c1\begin{pmatrix} -0.985 \\ 0.17 \end{pmatrix}e^{-0.69t} + c2\begin{pmatrix} 0.77 \\ -0.64 \end{pmatrix}e^{-0.145t} \tag{14}$$

In Eq. (14) c_1, c_2 are arbitrary constants. However, since it is an initial value problem, substitution of the initial values into the general solution allows calculation of c_1, c_2 which yields:

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 1.26 \\ -0.22 \end{pmatrix}e^{-0.69t} + \begin{pmatrix} -0.27 \\ 0.23 \end{pmatrix}e^{-0.145t} \tag{15}$$

The critical point's stability is shown in the phase diagram illustrated in Fig. 5, based on different initial values for market shares. As observed, whatever the initial conditions are, all trajectories

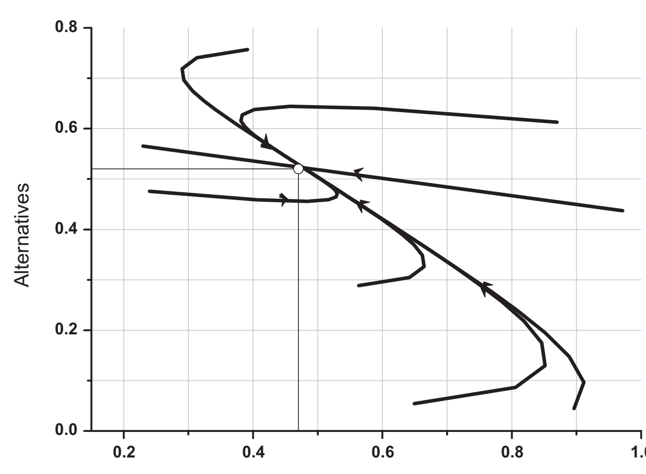

Fig. 5. Phase portrait of dynamic system based on random initial values for market shares. All trajectories tend to the critical point.

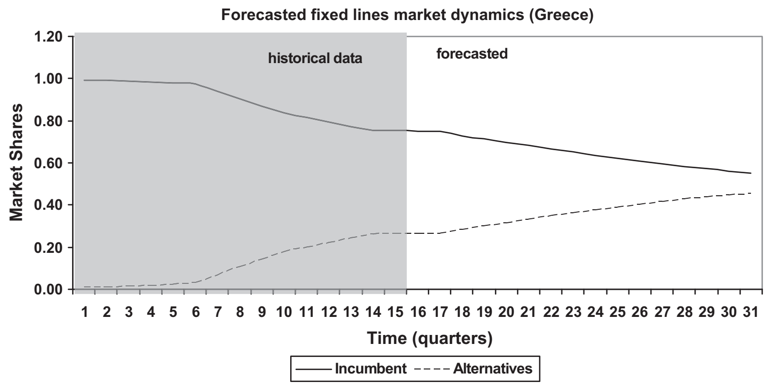

Fig. 6. Forecasted fixed lines market shares allocation, based on the estimated values of the constructed Lotka–Volterra model – Greece.

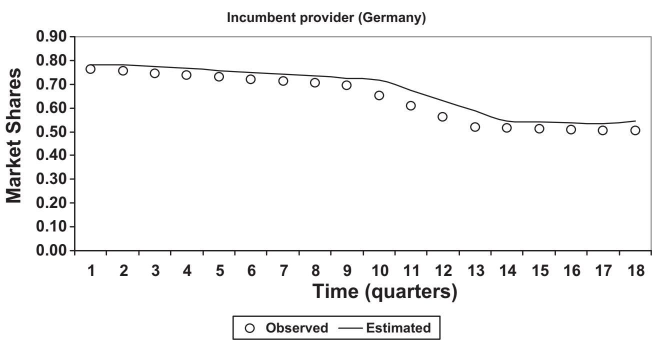

Fig. 7. Estimation results for incumbent operator's market shares – Germany.

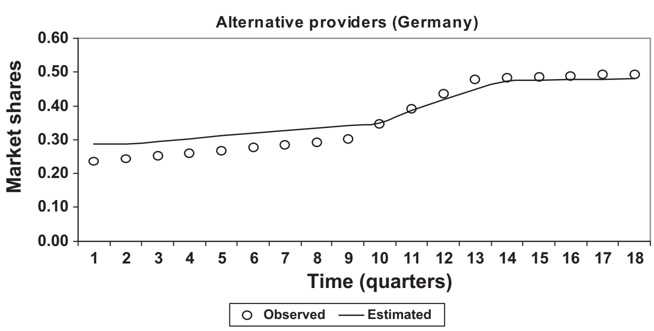

Fig. 8. Estimation results for alternative operators' market shares – Germany.

converge to the estimated critical point, because as time increases the solutions decay exponentially and approach the critical point.

Finally, Fig. 6 illustrates the expected market shares evolution, based on the system described by Eq. (12). The grey shaded portion of the graph refers to the historical data used to perform the evaluation and the white area to the forecasted values of market shares. The historical data corresponds to the time period starting from the first quarter of year 2001 up to the last quarter of year 2005.

The graph provides a measure of the rate of the subscribers switching between the incumbent provider and the alternatives. Although the calculated critical point is expected to be met in about eight years time, given the present dynamics of the market, the switching rate will increase rapidly during the next few years, at a value of about 1–2% per quarter.

The evaluation results for the rest cases considered are presented in the following subsections (Figs. 7–18). They were derived by applying the same procedure described so far, in the present section.

5.1. Evaluation results for Germany

See Figs. 7–9.

5.2. Evaluation results for Spain

See Figs. 10–12.

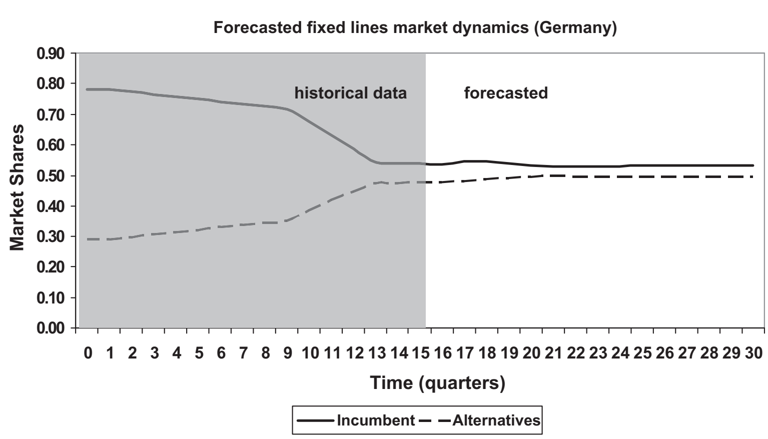

Fig. 9. Forecasted fixed lines market shares allocation, based on the estimated values of the constructed Lotka–Volterra model – Germany.

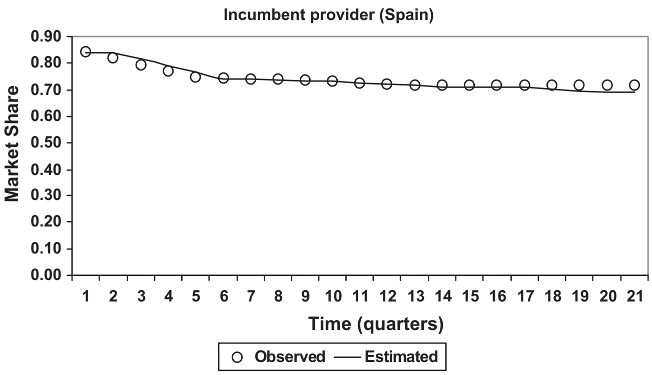

Fig. 10. Estimation results for incumbent operator's market shares – Spain.

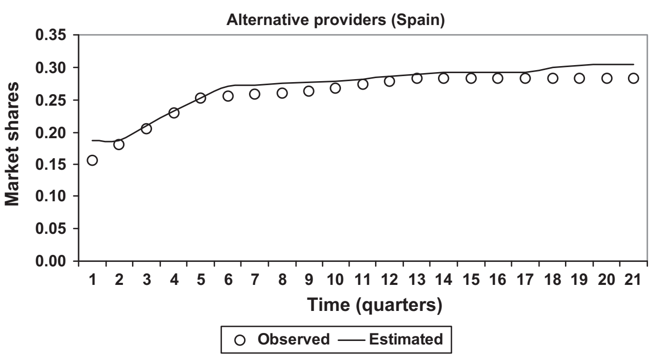

Fig. 11. Estimation results for alternative operators' market shares – Spain.

5.3. Evaluation results for United Kingdom

See Figs. 13–15.

5.4. Evaluation results for Ireland

See Figs. 16–18.

5.5. Results discussion

The results presented in this section reveal, together with the ability of the proposed model to describe market evolution, the different levels of competition between the incumbent and the alternative providers, across Europe. In addition, these changes in market shares reflect the influence of the pricing schemes over

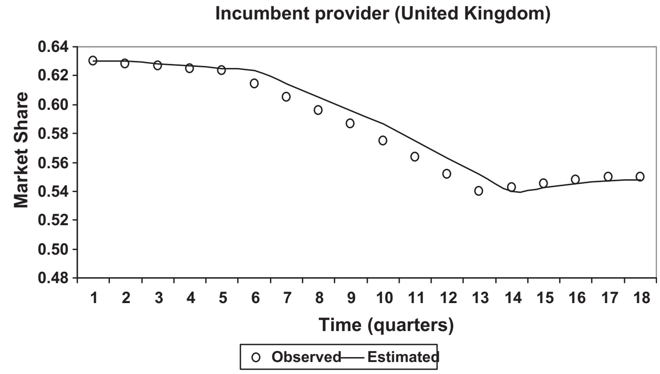

Fig. 13. Estimation results for incumbent operator's market shares – United Kingdom.

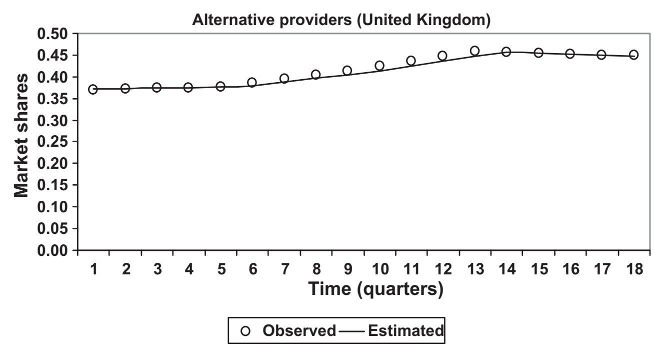

Fig. 14. Estimation results for alternative operators' market shares – United Kingdom.

the market shares (Eurostat, 2007) contains information that shows a continuous drop in prices regarding charges for national and international calls, together with charging differences between the incumbent provider and the competitors. Findings of the present evaluation are in accordance with this information, as pricing differences are depicted in the corresponding market shares. Indeed, in the cases where the incumbent is more expensive than the alternatives, its market shares decreased. However, by the time that a price reduction policy was adopted, it was interpreted into an increased or stable market share. The countries considered in this work also fall into this case. For example, BT in the United

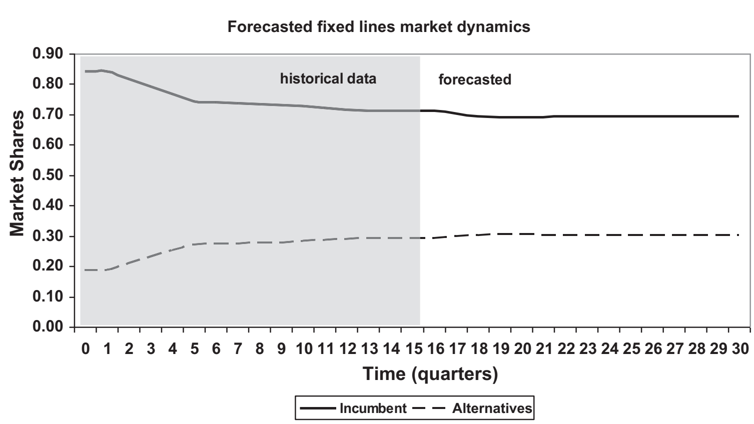

Fig. 12. Forecasted fixed lines market shares allocation, based on the estimated values of the constructed Lotka–Volterra model – Spain.

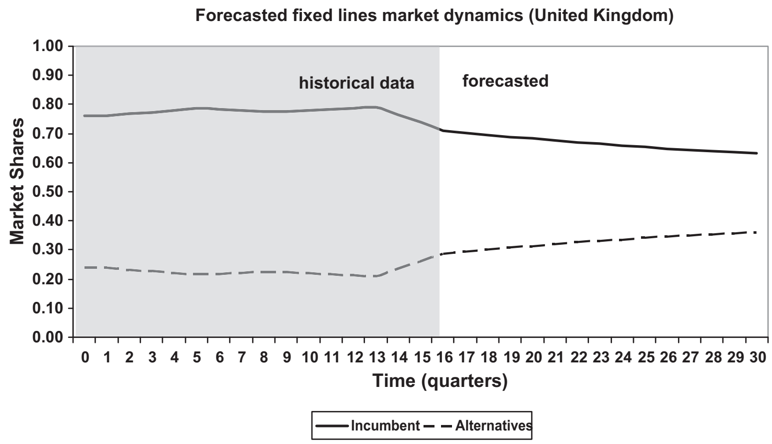

Fig. 15. Forecasted fixed lines market shares allocation, based on the estimated values of the constructed Lotka–Volterra model – United Kingdom.

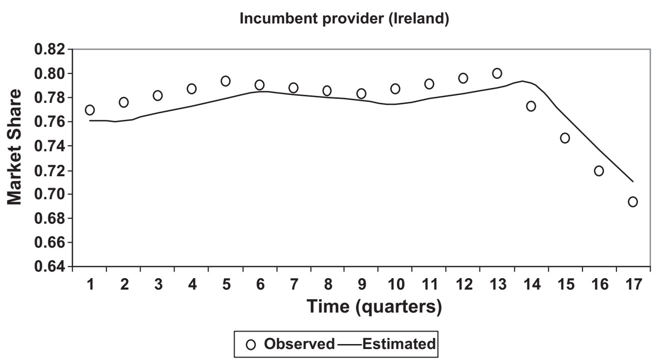

Fig. 16. Estimation results for incumbent operator's market shares – Ireland.

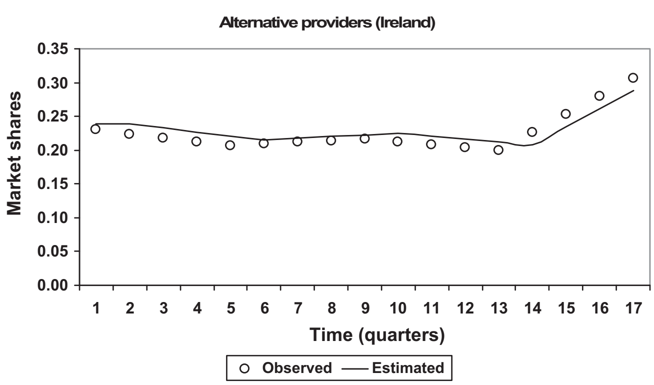

Fig. 17. Estimation results for alternative operators' market shares – Ireland.

Kingdom charged an almost double price for national calls, as compared to competition, until 2004. This was translated into a corresponding reduction in BT's market share. However, after 2004 when its prices for national calls where cheaper than competition, managed not only to retain its market share but also to increase it. However, this was not the case for Ireland, where Eircom, the incumbent operator, during 2003 and 2004 was charging national calls as a price more than double than its competitors and this was consequently translated into a substantial market share reduction. In Spain on the other hand, the incumbent operator adopted a pricing scheme almost the same as its competitors and it managed to have a high market share, which is expected to retain during the following years. The same considerations regarding charging hold for international calls as well, as can be observed in (Eurostat, 2007), which are also reflected into the findings of the present work.

6. Forecasting ability test

Testing of the proposed model's forecasting ability was based on using a portion of the dataset as a holdback sample for training the model, in order to attempt to forecast the remaining values that were held back. More specifically, the historical dataset was split into two parts, the "training" and the "holdback" data. The former part was used to train the model and estimate its parameters, whereas the latter was used to compare the actual recorded values with the ones provided by the model as forecasts. The training data includes the first quarter of year 2001 up to the second quarter of year 2003, leaving the rest quarters as the holdback sample for testing purposes. The selection of the training data portion was based on the fact that during that period of time, the rate of the "churn-effect" (switching from the incumbent to the alternatives) was quite high, thus testing the forecasting ability of the proposed model taking into account that period of time would provide useful information regarding the dynamics of the process.

Once again, the parameters of the system described by Eq. (9) were estimated by applying the genetic algorithms over the training dataset. Estimation and forecasting results are illustrated in Figs. 19 and 20 where the gray shaded portion of the graph refers to the forecasted values of the process and the white to the training portion of the dataset. The accuracy of fitting is quite high in this case as well, since the coefficient of determination (R^2) had a value of about 0.95 and the rest statistical measures were calculated at a value of 4.19E−04 for the Mean Squared Error (MSE) and a value of 0.123 for the Mean Absolute Percentage Error (MAPE). As a conclusion, the proposed methodology manages to capture the dynamics of the system quite early in time, providing valuable inputs for shaping decision making policies. This result indicates that the proposed methodology can be applied in similar cases of markets, providing useful information regarding their evolution.

7. Conclusion

The main target of this work is the proposal of a methodology, based on concepts of population biology, in order to describe the evolution of an already saturated high technology market having

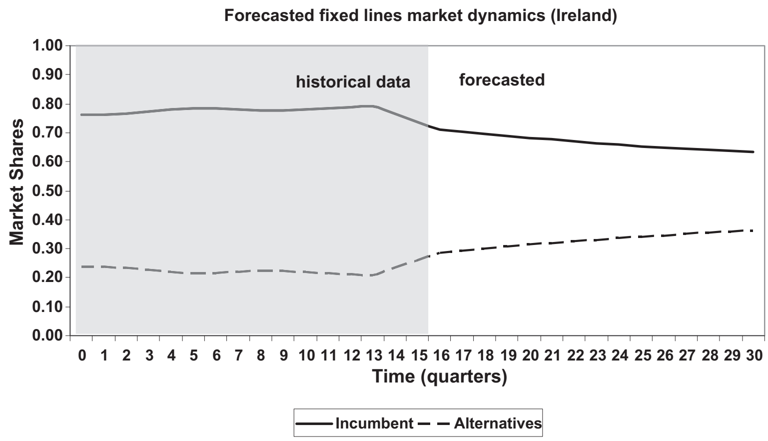

Fig. 18. Forecasted fixed lines market shares allocation, based on the estimated values of the constructed Lotka–Volterra model – Ireland.

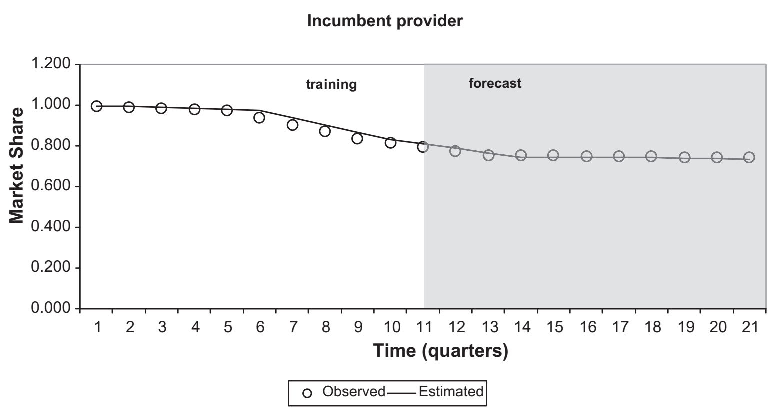

Fig. 19. Forecasted market shares for the incumbent provider, based on training data (years 2001–mid 2003).

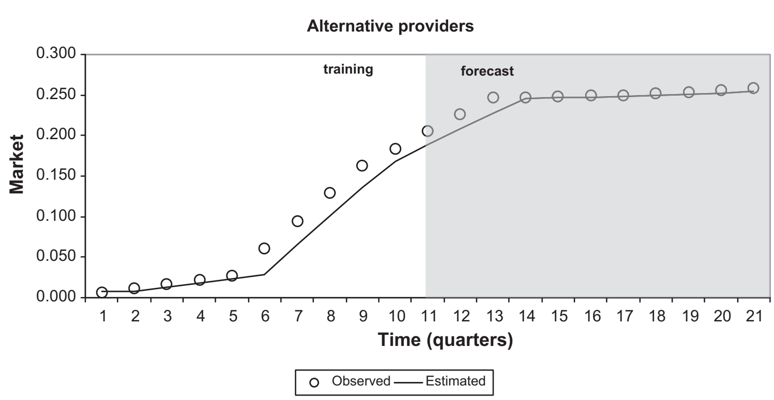

Fig. 20. Forecasted market shares for the alternative providers, based on training data (years 2001–mid 2003).

one dominant player, which the rest players have to compete with, in order to retain survival and growth. The mathematical description of the methodology was performed using the Lotka–Volterra model, in a prey–predator mode and the corresponding parameters were estimated by the means of genetic algorithms. The proposed methodology showed itself capable of estimating market equilibrium and market concentration and it can be applied over any high technology market meeting the above characteristics, providing valuable inputs for managerial decisions, strategic planning and regulatory decisions to the players of a high technology market.

As a case study, the structure of fixed lines market during the first stages after liberalization was evaluated and analyzed, over a number of European countries. The main assumption was to consider the incumbent and the alternative operators as prey and predator species, respectively, interacting for food supply. Evaluation results provided some very interesting information, regarding the estimation of the competition process and the market equilibrium. According to these, given the current process dynamics, the incumbent operator is expected to loose domination of the market if it does not respond quickly in order to create or to sustain a competitive advantage. Indeed, the evaluation of the proposed methodology provided results that reflect the influence of the pricing schemes over the corresponding market shares. Therefore, the incumbents can decide to divest activities and businesses that are characterized by low attractiveness and use the available financial resources to invest in new product–market combinations that create or sustain the competitive advantages of firms (van Kranenburg & Hagedoorn, 2008). Furthermore, customers are beginning to demand more telecommunications, information and media services in the form of one-stop-shopping (Graack, 1996). Due to changing demand, alternative companies have become more attractive to customers and businesses when they are able to deliver a critical mass of connected customers and content providers (Chacko & Mitchell, 1998).

Future work includes development of suitable framework of corresponding methodologies, based on the other approaches of Lotka–Volterra model as well, in order to comprehensively study the different aspects of the telecommunication market, into the context of the theory of population biology. Such examples include the application of the "competing species" approach, in order to describe the evolution of the market after it reaches its equilibrium and the symbiosis model, in order to describe the enhancement of the growth rate of adoption in the case of complementary products or services. Moreover, the presented methodology can be extended by itself, so as to separately consider each market competitor as a predator preying on the dominant player, in order to describe in a more detailed level the structure of the market, the degree of competition and the market equilibrium.

References

Banerjee, A., & Ros, A. J. (2004). Patterns in global fixed and mobile telecommunications development: A cluster analysis. *Telecommunications Policy, 28*(2), 107–132.

Baye, M. (2006). *Managerial economics and business strategy* (5th ed.). Boston: McGraw-Hill.

Begon, M., Townsend, C., & Harper, J. (2006). *Ecology: From individuals to ecosystems* (4th ed.). Blackwell Publishing.

Bewley, R., & Fiebig, D. G. (1988). A flexible logistic growth-model with applications in telecommunications. *International Journal of Forecasting, 4*(2), 177–192.

Boyce, W. E., & DiPrima, R. C. (2005). *Elementary differential equations and boundary value problems* (8th ed.). Hoboken, NJ: Wiley.

Chacko, M., & Mitchell, W. (1998). Growth incentives to invest in a network externality environment. *Industrial and Corporate Change, 7*(4), 731–744.

Curry, B., & George, K. D. (1983). Industrial Concentration - a Survey. *Journal of Industrial Economics, 31*(3), 203–255.

European Commission, (1987). Towards a Dynamic European Economy Green Paper on the development of the common market for telecommunications services and equipment, Corn (87), Brussels.

Eurostat. (2007). Consumers in Europe: Facts and figures on services of general interest.

Fay, T. H., & Greeff, J. C. (2008). A three species competition model as a decision support tool. *Ecological Modelling, 211*(1–2), 142–152.

Fisher, J. C., & Pry, R. H. (1971). A simple substitution model of technological change. *Technological Forecasting and Social Change, 3*(1), 75–88.

Freedman, H. I., & Waltman, P. (1977). Mathematical-analysis of some 3-species food-chain models. *Mathematical Biosciences, 33*(3–4), 257–276.

Freedman, H. I., & Waltman, P. (1984). Persistence in models of 3 interacting predator–prey populations. *Mathematical Biosciences, 68*(2), 213–231.

Freedman, H. I., & Waltman, P. (1985). Persistence in a model of 3 competitive populations. *Mathematical Biosciences, 73*(1), 89–101.

Goldberg, D. E. (1989). *Genetic algorithms in search, optimization, and machine learning*. Reading, Mass: Addison-Wesley.

Gompertz, B. (1825). On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. *Philosophical Transactions of the Royal Society of London, 115*, 513–585.

Graack, C. (1996). Telecom operators in the European Union: Internationalization strategies and network alliances. *Telecommunications Policy, 20*(5), 341–355.

Grzybowski, L. (2008). The impact of regulation on the retail prices in fixed-line telephony across the European Union. *Telecommunications Policy, 32*(2), 131–144.

Hartman, P. (1964). *Ordinary differential equations*. New York: Wiley.

Holland, J. (1975). *Adaption in natural and artificial systems*. Ann Arbor, Michigan: University of Michigan Press.

Kim, J., Lee, D. J., & Ahn, J. (2006). A dynamic competition analysis on the Korean mobile phone market using competitive diffusion model. *Computers & Industrial Engineering, 51*(1), 174–182.

Kirlinger, G. (1989). Two predators feeding on two prey species: A result on permanence. *Mathematical Biosciences, 96*, 1–32.

Leach, P. G. L., & Miritzis, J. (2006). Analytic behaviour of competition among three species. *Journal of Nonlinear Mathematical Physics, 13*(4), 535–548.

Lee, S., Lee, D. J., & Oh, H. (2005). Technological forecasting at the Korean stock market: A dynamic competition analysis using Lotka–Volterra model. *Technological Forecasting and Social Change, 72*, 1044–1057.

Li, J. X., & Yan, J. R. (2008). Partial permanence and extinction in an *N*-species nonautonomous Lotka–Volterra competitive system. *Computers & Mathematics with Applications, 55*(1), 76–88.

Lopez, L., & Sanjuan, M. A. F. (2001). Defining strategies to win in the Internet market. *Physica A, 301*(1–4), 512–534.

Lynch, S. (2007). *Dynamical systems with applications using mathematica*. Boston: Birkhäuser.

Marfels, C. (1972). Testing Concentration Measures. *Zeitschrift Fur Nationalokonomie, 32*(4), 461–486.

May, R. M., & Leonard, W. J. (1975). Nonlinear aspects of competition between three species. *SIAM Journal on Applied Mathematics, 29*(2), 243–253.

Michalakelis, C., Varoutas, D., & Sphicopoulos, T. (2008). Diffusion models of mobile telephony in Greece. *Telecommunications Policy, 32*(3–4), 234–245.

Modis, T. (1999). Technological forecasting at the stock market. *Technological Forecasting and Social Change, 62*, 173–202.

Murray, J. D. (2002). *Mathematical biology* (3rd ed.). New York: Springer.

Neal, D. (2004). *Introduction to population biology*. New York: Cambridge University Press.

Pao, C. V. (2003). Global asymptotic stability of Lotka–Volterra 3-species reaction–diffusion systems with time delays. *Journal of Mathematical Analysis and Applications, 281*(1), 186–204.

Rai, L. P. (1999). Appropriate models for technology substitution. *Journal of Scientific & Industrial Research, 58*(1), 14–18.

Saving, T. (Feb., 1970). Concentration Ratios and the Degree of Monopoly. *International Economic Review*, *11*(1), 139–146.

Seifert, G. (1995). A Lotka–Voltera predator–prey system involving two predators. *Methods and Applications of Analysis, 2*(2), 248–255.

Shair, A., & Tineo, A. (2007). Three-dimensional population systems. *Nonliear Analysis: Real World Applications, 9*, 1607–1611.

Shy, O. (1995). *Industrial organization, theory and applications*. New York: The MIT Press.

Tirole, J. (1988). *The theory of industrial organization*. Cambridge: The MIT Press.

Tschirhart, J. (2000). General equilibrium of an ecosystem. *Journal of Theoretical Biology, 203*(1), 13–32.

van Kranenburg, H. L., & Hagedoorn, J. (2008). Strategic focus of incumbents in the European telecommunications industry: The cases of BT, Deutsche Telekom and KPN. *Telecommunications Policy, 32*(2), 116–130.

Venkatesan, R., & Kumar, V. (2002). A genetic algorithms approach to growth phase forecasting of wireless subscribers. *International Journal of Forecasting, 18*(4), 625–646.

Wangersky, P. (1978). Lotka–Volterra Population Models. *Annual Review of Ecology and Systematics, 9*, 189–218.

Wijeratne, A. W., Yi, F., & Wei, J. (2007). Bifurcation analysis in the diffusive Lotka–Voltera system: An application to market economy. *Chaos, Solitons and Fractals*. doi:10.1016/j.chaos.2007.08.043.