Development of a demand model for the simultaneous estimation of the interaction between a product's diffusion and price: an application to telecommunications

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***Abstract*— The aim of this paper is the introduction of a new framework for demand estimation and forecasting along with a price index construction. The innovative concept developed is the "diffusion – price" model, an alternative long term estimation of price and diffusion elasticity, at the same time. It corresponds to the simultaneous estimation of the mutual influence of the product's price over its expected diffusion and vise versa. The discrete parts of the methodology are the use of a diffusion model for the initial estimation of diffusion, the construction of a price index function for estimating the pricing mechanism and, finally, the construction of a "diffusion-price" model for estimating and adjusting the diffusion level and price quantities. The evaluation process whose solution was based on genetic algorithms, revealed remarkable results which can be used for business strategies development, as the pricing policy is able to make diffusion diverge substantially from the initial estimates. The evaluation was performed over the DSL technology and the data correspond to the wider European area.**

***Index Terms*— Diffusion, forecasting, price indices, diffusion-price model, diffusion elasticity, price elasticity.**

I. Introduction

Telecommunications sector constitute a section of high technology area which is strongly related with rapid changes happening, more frequently than in any other market of products. The time needed for a newly introduced product to be substituted by its descendant generation or by another product is quite short, as compared to other products' durability. Thus, diffusion estimation and forecasting are of substantial interest, especially if the corresponding investments, which are usually related to the product's introduction, are considered. The accuracy of these estimations should be quite precise in order to depict the market's expected potential which in turn is the basic component for the business plans to be constructed. The present work targets exactly to this point; to make diffusion estimation process more accurate and provide indications for a proper pricing policy.

The question of acceptance or rejection of the product by the targeted potential adopters, or otherwise the consideration whether the product's expected diffusion will be successful or not, is a quite complicated aspect and can be considered as an output of a function of a number of decision variables. The most determinative ones are the market structure, from the standpoint of competition, the product's marketing strategy and, of course, its price. Marketing strategy together with pricing are strongly related to the influence of the product's life cycle main characteristics, such as market potential and period of existence. Carefully planned marketing strategies, such as product bundling or tying can sustain its life time as well as its initially estimated saturation level.

Price is probably the most important factor determining the product's life cycle, from its take off to its ceasing to exist stage. Most technological innovations are introduced immediately after this is technically feasible, however at relative high prices, which discourage the potential adopters from immediately accepting the products, even if they may correspond to radical technological breakthroughs. However, this situation is usually reverted, following price reductions, which make adoption affordable. In an intuitively way of thinking, diffusion process can be accelerated if suitable pricing policies are adopted, which in turn influence pricing, as the more the adopters, the easier for the supplier to set prices closer to its marginal cost of the product's production.

This last consideration constitutes the main contribution of the present work, as it attempts to describe the way diffusion and pricing are expected to interact, thus adjusting initial estimations for both quantities. The proposed methodology attempts to fill the gap that exists in the diffusion theory, where the corresponding models consider mainly the time as the only diffusion variable, into which the underlying diffusion mechanics are accommodated.

It should also be pointed that the diffusion process of a high technology product is influenced by factors which can be located outside the referenced market. The most characteristic of them is the influential pressure of the adoption level of the neighboring countries or the adjacent markets, in general [1].

The present work is formed as follows: Section II presents the basic concepts of the theoretical background of diffusion theory and models and price indices construction. Section III is devoted to the development of the introduced methodology, together with an overview of genetic algorithms, which are used to solve the set of equation participating into the proposed methodology. Section IV presents an evaluation case of the methodology, over the ADSL technology diffusion and pricing across the European countries' area, as well as the analysis of the results. Finally, Section V provides some useful conclusions, and directions for future research.

II. THEORETICAL BACKGROUND

Diffusion models

Diffusion models are mathematical functions of time, used to estimate the parameters of the diffusion process of a product's life cycle, usually at an aggregate level, without taking in consideration the underlying specific parameters that drive the process.

The most well-known representatives of the models developed for diffusion estimation, are the Bass model [2], the Fisher – Pry model [3], the logistic family models [4], as well as the Gompertz model [5].

Logistic models and variations of the Gompertz model provide S- shaped curves, which are commonly used in forecasting diffusion of products or services. These models are used to describe and forecast demand and diffusion at the aggregate level, which is the total market response rather than at the individual customer level (this approach is described by the so called choice-based models focusing on the probability of individuals to adopt the innovation whose market behavior is driven by maximization of preferences, as modern economic choice theory assumes). S-shaped patterns derive from the differential equation:

$$\frac{dY(t)}{dt} = \delta * Y(t) * [S - Y(t)] \tag{1}$$

In Eq. (1), Y(t) represents the total penetration until time t, S the saturation level of the specific technology and δ is a constant of proportionality, the so-called coefficient of diffusion. Penetration is defined as the proportion of the population that uses the product or service being examined.

At the time that the particular technology is introduced (t=0), there is a critical mass, the innovators that initially adopt it. This number influences the rate of diffusion and the time of saturation is met.

In the context of this work, the Linear Logistic Model is used in order to estimate the penetration of the technology at each year.

The general form of the logistic models family is:

$$Y(t) = \frac{S}{1 + e^{f(t)}} \tag{2}$$

where Y(t) is the estimated diffusion level and S the saturation level.

f(t) is given by the following formula:

$$f(t) = -a - b * t(m,k) \tag{3}$$

where t(m,k) is a non-linear function of time (except the linear logistic model, where t(m,k)=t) and is given by one of the following formulations, according to the model's construction.

The Linear instance of the model is given by

$$t(m,k) = t \tag{4}$$

The linear logistic model is also known as Fisher - Pry model [3].

Diffusion process studies are of paramount importance towards the understanding of the mechanism that drives the acceptance or rejection of a product or service from the potential adopters, within a market. This consideration has a field of reference the newly introduced products, as much as the established ones.

As far as the study of the impact of decision variables into the diffusion process is concerned by including them into the corresponding models, some important work has already been introduced in literature [6], [7]. However, the added value of the presented work is that the decision variable participates into the constructed model not simply as a recorded value, but as an output of a function based on the corresponding theory of price indices and hedonic functions. In this way, the two functions can interact between each other and their interaction can be quantified and consequently used in other calculations, such as the construction of pertinent reaction functions.

Hedonic Functions and Price Indices

Hedonic methods refer to regression models in which product prices are related to product characteristics and the observed price of a product (service) is considered to be a function of its characteristics. Generally hedonic methods are based on the idea that a service (product) is a bundle of characteristics and that consumers just buy bundles of product characteristics instead of the product itself. These methods can be used to construct a quality-adjusted price index of a service. Triplett [8] described an overview on hedonic price equations. Rosen [9] states that from a large amount of product varieties, consumer chooses without influencing prices. Therefore, consumers maximize utility and producers maximize profits. In hedonic studies it is possible to adjust the price of a service for its quality and not its quantity.

Regarding the construction and study of price indices for the ADSL technology services, [10, 11] refer to corresponding work carried out.

The term "hedonic methods" is related to the use of a "hedonic function" *f(X)* use in economic measurement, where

$$P_i = f(X_i) \tag{5}$$

with P_i being the price of a variety (or a model) *i* of a product and X_i is a vector of characteristics associated with the variety. The hedonic function is then used, for different characteristics among varieties of the product, in calculating the price index.

A typical example of a hedonic function is the following:

$$ln(P_i) = b_0 + b_1 X_{1i} + b_2 X_{2i} + \ldots + b_k X_{ki} + u_i \qquad i = 1, 2, \ldots, N \tag{6}$$

In Eq. (6) *P* is the value of the *i-th* recorded value for the price of a product with a number of *k* characteristics that influence the observed price. This constitutes a simple example, where the product's characteristics contribute linearly to the construction of the hedonic function. In other cases of hedonic functions the included characteristics can contribute in other forms, such as logarithmic, exponential or in a sinusoidal form. The b_i's correspond to the parameters that have to be estimated for the model to be completed. Based on the estimated hedonic function, the corresponding price index is constructed in a way that reflects the change in price between two subsequent time periods. Finally, the terms u_i correspond to the regression residuals.

III. THE "DIFFUSION-PRICE" MODEL

Diffusion

Considering the generic diffusion equation expressed by Eq.(1), the incorporation of a decision variable, such as price leads to the following equation:

$$\frac{dF(t)}{dt} = d * F(t) * [S - F(t)] * x(t) \tag{7}$$

In Eq.(7), $x(t)$ is the current marketing effort term which includes those effects that are happening at time t and influence the adoption rate. In order to model the impact of price, $x(t)$ is modeled as

$$x(t) = 1 + (k * \textit{change at time t in price}) \tag{8}$$

In Eq.(8), 1 represents the natural time and k measures the impact and corresponding adjustment, of price over diffusion. This can be represented by:

$$x(t) = 1 + (k * \frac{dP(t)}{dt}) \tag{9}$$

P(t) corresponds to the price at time t, which is estimated by the means of a suitably constructed price index. Thus, the model that describes the influence of price in the diffusion process becomes:

$$F(t) = S * \frac{1}{1 + e^{-\alpha - \beta*(t + k*P(t))}} \tag{10}$$

Eq. (10) captures the cumulative diffusion by simultaneously incorporating the influence of price which is depicted by the factor *k*P(t)* of the equation, along with the time variable, t, which accumulates all the underlying mechanics that drive the diffusion process.

Hedonic Function

The construction of a price index methodology based on hedonic functions was presented in Section II and expresses the relation between the prices of different varieties of a product, in correspondence with their characteristics. However, this approach misses to capture the effect of demand over the pricing policy. More specifically, by the time new technology products are introduced they are offered in relatively high prices. These prices usually continue to stay at this high level during the introduction stage and they are subsequently lowered, following the advance of technology and the production cost, the competitiveness of the market of reference and, of course, the corresponding acceptance of the product, expressed as the demand for it. This last point, the level of diffusion during a period of time, can consequently influence the suppliers' pricing policy for the product. This holds true especially in the area of telecommunications, where products and services belong to the group of network products, from the economic point of view. Thus, the more the adopters of the product the higher the utility gained, which is a crucial factor for a successful life cycle. If combined with a corresponding suitable pricing policy it can strengthen the expectations of acceptance in the targeted market.

Based on these considerations, the proposed model incorporates the reaction of pricing process to the change of diffusion. It is based on the same assumption as in the previous case of diffusion, although in the opposite direction.

The incorporation of the influence of diffusion in the function that describes a product's price leads to the following Eq.(11)

$$P = f(t, F(t), X_i) + \varepsilon \tag{11}$$

In Eq.(11), P is the estimated price, which is expressed as a function of time, t, diffusion level of the product at that point of time F(t) and of course the rest of the product's characteristics that influence its price. ε stands for the estimation error. For the evaluation case performed in this work, over diffusion and pricing of the ADSL technology services, the following price function is used:

$$ln(P) = e^{a*ln(D) + b*ln(U) + c*(t + l*F(t)) + d} + \varepsilon \tag{12}$$

Eq.(12) is in accordance with the general form of a hedonic function, as defined in Eq.(11). The participating variables are *D*, to denote the Downlink speed, *U* for the Uplink speed, and *t* and *F(t)* to stand for the time variable and the cumulative diffusion at time t respectively. *a, b, c, l* and d are the model's parameters and, finally, ε is the estimation error. For simplicity reasons the *ln(P)* is used to represent the more formal $ln(P(t))$, which denotes that price, among other variables is a function of time as well.

The choice of the model can be based on either some statistical measures of accuracy, such as Mean Square Error (MSE), Mean Absolute Percentage Error (MAPE) etc. The choice among the candidate models can be made according to the values of the statistical measure each model reveals during evaluation. An alternative approach is to consider a theoretical criterion for choosing the most appropriate model, such as the Akaike Information Criterion (AIC) [12, 13]. Finally, it should be noted that the $P(t)$ part in Eq.(10) is the $ln(P)$, as introduced in Eq.(12).

Solution Algorithm

The set of Eq. (10) and (12) are solved simultaneously, in an iterative way, by following the next steps [14].

1. Assign a value of 0 to $ln(P), F(t)$ on the right-hand side of Equations (10) and (12), respectively.
2. Estimate $(\alpha,\beta, S, a, b, c, d)$ of the two resulting equations. Call them $(\alpha,\beta, S, a, b, c, d)_0$
3. Using $(\alpha,\beta, S, a, b, c, d)_0$ and using 0 for $ln(P), F(t)$ on the right-hand sides, evaluate $F(t), ln(P)$ of Equations (10) and (12). Call these $(F(t), ln(P))_1$.
4. Assign $(F(t), \ln(P))$ to the $F(t)$ and $ln(P)$ on the right-hand side of Equations (10) and (12) and estimate $(\alpha,\beta, S, a, b, c, d)$. Call them $(\alpha,\beta, S, a, b, c, d)_1$.
5. Using $(\alpha,\beta, S, a, b, c, d)_1$ and using $(F(t), ln(P))_1$ for $F(t)$ and $ln(P)$ on the right- hand sides, evaluate $(F(t), ln(P))$ of Equations (10) and (12). Call these $(F(t), ln(P))_2$.
6. Assign $(F(t), ln(P))_2$ to $F(t)$ and $ln(P)$ on the right-hand side of Equations (10) and (12) and estimate $(\alpha,\beta, S, a, b, c, d)$. Call them $(\alpha,\beta, S, a, b, c, d)_2$
7. Repeat Steps 5 and 6 until no changes in the estimates of $(\alpha,\beta, S, a, b, c, d)$ are found.

The above procedure is implemented by using a genetic algorithms approach, as an analytical solution is not available. The objective function for the algorithm was the minimization of the squares of the errors, between the actual and the estimated values of both penetration and price.

III. EVALUATION OF THE PROPOSED MODEL

Data

As already mentioned, the evaluated case of this work is the ADSL technology. The data correspond to the mean values of the ADSL penetration and price characteristics across the EU15 countries. Obviously, the developed methodology can also be applied over more specific cases, such as individual countries, or otherwise defined markets. The data used in this work were chosen in order to depict an overview of the mutual influence of penetration and price over an area of general interest, such as the European area.

Table 1 Diffusion of the ADSL in the EU15 area (Source: Eurostat)

Year	Penetration
2001	2,42
2002	5,013
2003	8,213
2004	12,633
2005	17,753

Table 2 Mean values for price and bandwidth characteristics for ADSL access across the EU15 countries (Source: ADSL Providers)

Year	Price (€)	Downlink (Kbps)	Uplink (Kbps)
2003	80,16	1062,35	185,22
2004	44,57	2045,88	606,88
2005	48,58	3591,74	840,91

Table 1and Table 2contain the mean values for the data used for the present evaluation of the "diffusion-price" model. To perform the initial estimation of the price index, the full dataset for ADSL prices and bandwidth characteristics was used, which is available in Appendix For estimating the impact of price on diffusion the Ceteris-Paribus principle was adopted, according to which the Downlink and Uplink values of an ADSL connection were kept constant, whereas the time was the only variable to change.

Results

The evaluation procedure provided the results are presented in this section. Table 3 contains the numerical results, which are graphically depicted in Figure 1and Figure 2. In these figures, the "Converged" index corresponds to the results of Iteration2. The convergence of the algorithm was achieved at the second iteration, after the genetic algorithms have created a number of about 500.000 population sizes, at each step. The values estimated during Iteration 1, do not have any other special physical meaning than that they are the intermediate step towards convergence, since new parameters and variables are introduced into the initial model.

Table 3 Estimation results of the "diffusion-price" model

	Diffusion		Price	
Initial Estimates	S	29,620	a	-0,068
	α	-3,003	b	-0,014
	β	0,680	c	-0,003
			d	1,964
Iteration1	S	29,6204366	a	-0,068
	α	-3,0031295	b	-0,01419
	β	0,68018621	c	-0,00276
			d	1,9636
	k	-0,67	l	-0,34
Iteration2	S	50,1860703	a	-0,068
	α	-1,8241095	b	-0,01419
	β	0,49907036	c	-0,00276
			d	1,9636
	k	-0,6153329	l	-0,0574

The values of k and l, as these were calculated during Iteration 2, represent the finally estimated proportionality of influence. More specifically, k indicates that a reduction in price contributes to an increment in diffusion by a factor of 0,6153 its value. The minus sign of k has the meaning that there is a converse proportionality between diffusion and price, as expected. The same evidence holds for l, the factor of influence of diffusion over price. An interesting point is that, as intuitively expected, the influence of price over demand is quite higher than the opposite direction. It is obvious that by setting a lower price demand increases, but the vice versa can be the case only for the amount that makes the marginal cost of production equal the marginal profit.

In the present case study, evaluation results revealed that the saturation level of demand (S) is expected to reach a substantially higher level than initially estimated, if the interaction with the corresponding pricing policy is considered. Thus, for a given set of Downlink and Uplink connection characteristics, the pricing path declines over time and at the same time diffusion increases more rapid than calculated by employing a diffusion model.

The main contribution of the methodology is that the examination of the reaction functions of demand and price can lead to the estimation of the changes that are expected to happen in both of the participating quantities. Thus, the "diffusion-price" model can be a helpful tool towards the construction of a corresponding business plan, as it indicates the pricing policy that should be followed.

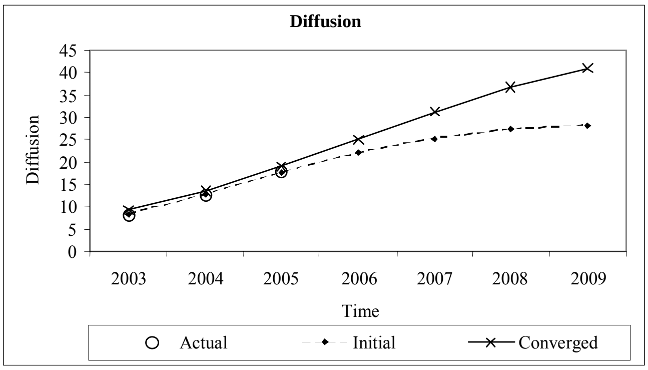

Figure 1 Influence of price over diffusion of the ADSL technology.

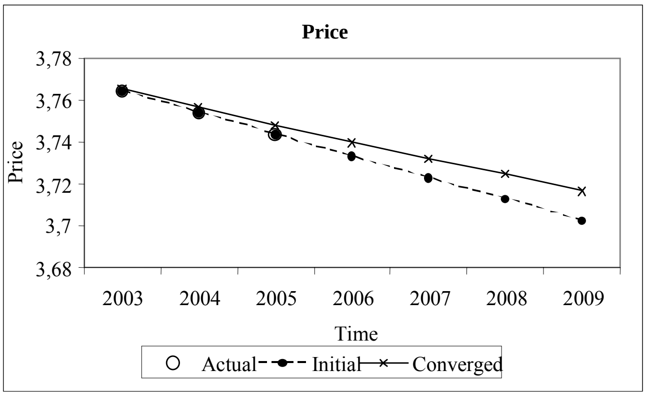

Figure 2 Influence of diffusion over the expected pricing schema

IV. Conclusions

In this paper, a "diffusion-price" is introduced, in the context of a corresponding methodology, for incorporating the effect of a product's price over its estimated and forecasted adoption rate and vice-versa. The developed approach extends the classical diffusion theory where the majority of diffusion models consider the elapsed time since the introduction of a product as the only diffusion variable. However, they cannot always succeed in capturing the influence of the parameters that drive the adoption process and its characteristics, such as the product's saturation level. For this reason, the proposed methodology makes an important contribution as it can provide a tool for estimating the expected price and demand elasticity at the same time, and the way they both interact.

Moreover, in this work price participates not merely as recorded values, but as an output of a methodologically derived process, based on price indices and hedonic functions.

Future directions for research may include the study of incorporating other substantially influential decision variables, such a marketing force and the consequent affection of the demand process.

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V. REFERENCES

[1] C. Michalakelis, G. Dede, D. Varoutas, and T. Sphicopoulos, "Impact of cross-national diffusion process in telecommunications demand forecasting," presented at NAEC 2005, Garda, Italy, 2005.

[2] F. M. Bass, "A new product growth model for consumer durables," *Management Science*, vol. 15, pp. 215-227, 1969.

[3] J. C. Fisher and R. H. Pry, "A simple Substitution Model of Technological Change," *Technological Forecasting and Social Change*, vol. 3, pp. 75-88, 1971.

[4] R. Bewley and D. G. Fiebig, "A Flexible Logistic Growth-Model with Applications in Telecommunications," *International Journal of Forecasting*, vol. 4, pp. 177-192, 1988.

[5] L. P. Rai, "Appropriate models for technology substitution," *Journal of Scientific & Industrial Research*, vol. 58, pp. 14-18, 1999.

[6] F. M. Bass, T. V. Krishnan, and D. C. Jain, "Why the Bass Model Fits without Decision Variables," *Marketing Science*, vol. 13, pp. 203-223, 1994.

[7] R. M. Heeler and T. P. Hustad, "Problems in Predicting New Product Growth for Consumer Durables," *Management Science*, vol. 26, pp. 1007-1020, 1980.

[8] J. E. Triplett, *"Hand book on hedonic indexes and quality adjustments in price indexes"*: OECD publishing, 2004.

[9] S. Rosen, "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition," *Journal of Political Economy*, vol. 92, pp. 34-55, 1974.

[10] N. Deligiorgi, A. Vavoulas, C. Michalakelis, D. Varoutas, and T. Sphicopoulos, "Factors and characteristics that affect ADSL tariffs," presented at 5th Conf. Telecom TechnoEconomics (CTTE2006), Athens, Greece, 8-9 June, 2006.

[11] N. Deligiorgi, A. Vavoulas, C. Michalakelis, D. Varoutas, and T. Sphicopoulos, "Nonparametric Estimation of a hedonic price index for ADSL connections in the European market using the Akaike Information Criterion," presented at Networking and E-Commerce Conference, NAEC 2006, Lake Garda, Italy, Oct. 2006.

[12] J. S. Simonoff and C. L. Tsai, "Semiparametric and additive model selection using an improved Akaike information criterion," *Journal of Computational and Graphical Statistics*, vol. 8, pp. 22-40, 1999.

[13] P. A. Naik and C. L. Tsai, "Single-index model selections," *Biometrika*, vol. 88, pp. 821-832, 2001.

[14] V. Kumar and T. V. Krishnan, "Multinational diffusion models: An alternative framework," *Marketing Science*, vol. 21, pp. 318-330, 2002.

VI. APENDIX A – FULL ADSL PRICE DATASET

Year	Country	Price (€)	Downlink (Kbps)	Uplink (Kbps)
2003	Belgium	55,88	3000,00	182,86
	France	56,57	749,71	109,71
	Germany	30,44	1472,00	192,00
	Italy	57,09	484,60	147,20
	Netherlands	58,32	853,33	149,33
	Spain	92,47	930,67	185,33
	Switzerland	155,63	902,74	322,53
	UK	132,08	835,33	250,00
	Greece	83,00	332,80	128,00
2004	Austria	45,32	1186,91	215,27
	Belgium	37,40	2599,20	224,00
	Denmark	58,57	1467,73	302,93
	Finland	50,69	1996,80	512,00
	France	28,93	1896,44	245,33
	Germany	34,55	1917,80	242,40
	Iceland	40,18	2048,00	512,00
	Italy	22,36	3030,00	2131,00
	Netherlands	45,36	2832,80	568,00
	Norway	41,52	1594,44	550,81
	Spain	87,79	1345,78	258,67
	Sweden	38,45	3978,56	2252,89
	Switzerland	49,42	1500,00	250,00
	UK	43,52	1247,81	231,06
2005	Austria	51,18	1808,57	1332,57
	Belgium	42,09	5534,29	353,14
	Denmark	44,09	1676,00	286,00
	Finland	36,62	3245,54	600,65
	France	30,65	7469,87	939,73
	Germany	29,33	2672,00	329,81
	Iceland	88,27	3072,00	512,00
	Italy	22,17	4391,27	3006,55
	Netherlands	46,10	4668,82	681,36
	Norway	68,11	5323,41	649,59
	Spain	79,24	1613,60	245,60
	Sweden	37,36	7788,00	2820,89
	Switzerland	49,32	1500,00	250,00
	UK	44,14	1792,00	291,52
	Greece	60,06	1320,73	314,18